ПЕДАГОГІЧНА АКАДЕМІЯ:
НАУКОВІ ЗАПИСКИ

TEACHING THEORY AND METHODOLOGY

УДК: 37.091.33:51
DOI https://doi.org/10.5281/zenodo.20362689

A three-layer model for understanding mathematical text: thesaurus – semantics – semiotics

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Прийнято: 12.04.2026 | Опубліковано: 30.04.2026

Abstract: *One of the main sources of acquiring mathematical knowledge in higher education is mathematical text. However, many students experience significant difficulties in reading and understanding mathematical texts. These difficulties are associated with insufficient knowledge of terminology, an inability to identify semantic relationships between mathematical concepts, and problems in interpreting various forms of representation of mathematical symbols and information.*

This article proposes a three-layer model for understanding mathematical text based on the integration of the thesaurus approach, semantic analysis, and the theory of semiotic registers. The model describes the process of forming a student's system of knowledge while working with mathematical text.

According to the proposed model, understanding mathematical text is formed at the following three interconnected levels:at the semiotic level – interpreting the forms of representation of mathematical information; at the semantic level – identifying the logical relationships between mathematical concepts; at the thesaurus level – forming a system of mathematical knowledge.

***The results** obtained may serve as a methodological basis for developing students' skills in working with mathematical information in the course of mathematical analysis.*

***Keywords:** mathematical text, thesaurus approach, semantic relations, semiotics, mathematical analysis, educational information.*

Трирівнева модель розуміння математичного тексту: тезаурус – семантика – семіотика

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Анотація: *Дослідження спрямоване на розробку інтегративної моделі формування розуміння математичного тексту у студентів вищих навчальних закладів. Актуальність роботи обумовлена труднощами, з якими стикаються студенти при читанні та інтерпретації математичних текстів, що включають визначення, теореми, доведення, формули та графічні зображення.*

Методи. *У дослідженні використано теоретичний аналіз психолого-педагогічної та методичної літератури, концептуальне моделювання, а також порівняльний аналіз існуючих підходів (семіотичного, тезаурусного та когнітивного).*

Результати. *Запропоновано трирівневу модель розуміння математичного тексту, яка включає: семіотичний рівень (інтерпретація форм подання інформації); семантичний рівень (встановлення логічних зв'язків між поняттями); тезаурусний рівень (формування системи математичних знань). Доведено, що ефективне розуміння математичного тексту можливе лише за умови взаємодії цих трьох рівнів.*

Висновки. *Розроблена модель може бути використана як методологічна основа для формування у студентів умінь працювати з математичною інформацією в курсі математичного аналізу.*

Ключові слова: *математичний текст, семіотика, семантика, тезаурус, математичний аналіз*

In mathematics education, mathematical knowledge is transmitted to students primarily through mathematical texts. Such texts consist of various elements:

definitions, theorems, proofs, rules and algorithms, examples, graphs, figures, and explanations. The totality of these elements constitutes the logical structure of mathematical knowledge.

A student's ability to work with mathematical text is an important condition for the successful mastery of mathematical disciplines. However, teaching experience and existing research show that students often encounter significant difficulties in reading and understanding mathematical texts [7].

The most common difficulties include the following [13]:

- inability to distinguish mathematical terms;
- failure to understand the differences between definitions and theorems;
- inability to correctly identify the logical relationships between concepts;
- misinterpretation of mathematical symbols.

In the methodology of mathematics teaching, there are various theoretical approaches aimed at explaining the process of understanding mathematical information on a scientific basis.

For example, R. Duval emphasizes that mathematical thinking is closely related to the use of different semiotic registers, that is, different forms of representation [3]. According to his theory, understanding mathematical objects develops through transformations between different forms of representation.

APOS theory explains the cognitive process of mathematical concept formation and describes the development of mathematical knowledge through the sequence of action, process, object, and schema [1].

In addition, the thesaurus approach plays an important role, according to which knowledge is viewed as a system of interconnected concepts [12, 16].

Despite the existence of these theoretical approaches, integrative models describing the process of understanding mathematical text in contemporary mathematics education methodology remain insufficiently developed.

ISSN: 2786-9458

The aim of this study is to develop a three-layer integrative model for understanding mathematical text based on the integration of thesaurus, semantic, and semiotic approaches.

2. Theoretical Foundations of the Study

2.1. The Concept of Mathematical Text

Mathematical text is a system of logically connected elements through which mathematical knowledge is conveyed.

The main elements of mathematical text include definitions, theorems, proofs, rules and algorithms, formulas, problems, graphs, and figures. Each of these elements performs a specific function within the structure of mathematical knowledge.

For example, definitions introduce new mathematical concepts, theorems express the properties of mathematical objects, and proofs substantiate the truth of these properties.

Thus, mathematical text constitutes a structured system of knowledge.

2.2. The Thesaurus Approach in Mathematics Education

The thesaurus approach considers knowledge as a system of concepts and the relationships between them [12].

Within this approach, mathematical knowledge can be represented as a conceptual network consisting of mathematical concepts, logical relations between them, and a hierarchy of concepts. The formation of a mathematical thesaurus helps to build a system of interconnected concepts in the student's mind [10, 15].

For example, when studying the topic of bounded sets, the following system of concepts is formed in students' vocabulary: set, upper bound, lower bound, supremum, infimum, bounded set.

2.3. Semantic Relations Between Mathematical Concepts

Relations between concepts are multifaceted [15], and from the perspective of mathematics teaching methodology, it is advisable to identify the most essential ones.

 ISSN: 2786-9458

The main types of such relations include: genus–species, part–whole, cause–effect, precondition, equivalence, and counterexample/negation.

2.4. Semiotic Registers in Mathematics

According to Duval's theory, mathematical information is represented in different forms [3]. The main registers are presented below (Table 1).

Table 1. Main Registers

Register	Example
Verbal	definition text
Symbolic	mathematical formulas
Graphical	coordinate axis
Tabular	table of values

According to R. Duval's theory, understanding a mathematical object is formed through transformations of information across registers.

3. Research Methodology

The research is based on theoretical and conceptual analysis methods.

The main objectives of the study were defined as follows:

- to analyze the structure of mathematical texts in the course of mathematical analysis;
- to identify mathematical concepts and their semantic relationships;
- to analyze various forms of representation of mathematical information;
- to develop an integrative didactic model.

4. Research Results

4.1. Structure of Mathematical Text

The analysis showed that mathematical text usually has the following structure:

terms → definitions → theorems → proofs → examples

This sequence reflects the logical development of mathematical knowledge.

4.2. Semantic Network of Concepts

Mathematical concepts form a semantic network.

 ISSN: 2786-9458

For example:

Set E

↓

upper bound

↓

supremum

4.3. A Three-Layer Model for Understanding Mathematical Text

Based on the analysis, the following model was developed.

1. **Semiotic layer** – this layer is associated with the interpretation of different forms of representation of mathematical information. These include textual descriptions, mathematical symbols, and graphical images.

2. **Semantic layer** – at this level, logical relationships between mathematical concepts are established.

3. **Thesaurus layer** – at this level, the student's system of mathematical knowledge is formed.

5. Discussion

The obtained results make it possible to consider the process of understanding mathematical text as a complex cognitive process involving several interconnected levels of information processing. The proposed model shows that understanding mathematical text is not limited to interpreting symbols or mastering mathematical concepts. Rather, it develops through the interaction of several cognitive mechanisms.

First, the interpretation of symbolic forms of mathematical information plays an important role. According to R. Duval's theory of semiotic registers, mathematical thinking is impossible without transformations between various forms of representation, such as symbolic notation, graphical representation, and verbal description. The ability to transform information across different registers is one of the main conditions for understanding mathematical objects [3].

 ISSN: 2786-9458

Second, identifying semantic relationships between mathematical concepts is of great importance. Mathematical knowledge is organized not as a collection of isolated facts, but as a system of interconnected concepts and statements. Therefore, understanding mathematical text includes the ability to identify logical relationships between mathematical objects. Research on cognitive processes in mathematics emphasizes that constructing such relationships is an important element of mathematical thinking [4, 11].

Third, understanding mathematical text is connected with the formation of a system of mathematical concepts, or a mathematical thesaurus. Within the framework of the thesaurus approach, knowledge is viewed as a network of interconnected concepts. The formation of such a network ensures a holistic understanding of mathematical content.

The integration of these three aspects makes it possible to consider understanding mathematical text as a multi-level cognitive process. Unlike existing approaches, which usually focus either on the semiotic aspects of mathematical thinking or on the cognitive mechanisms of concept formation, the proposed model integrates several theoretical directions in contemporary mathematics education methodology.

The obtained results are consistent with studies devoted to students' reading of mathematical texts. For example, studies by M. Inglis and L. Alcock show that students and experts use different strategies when reading mathematical proofs [5, 6, 8]. Experts pay more attention to the structure of proofs and the relationships between mathematical statements, whereas students often focus on individual symbolic expressions.

Similar results were obtained in K. Weber's studies on strategies for reading mathematical proofs. These studies show that successful understanding of mathematical text requires active analytical work, including the identification of logical relationships and the interpretation of mathematical symbols [14].

ISSN: 2786-9458

Thus, the research results confirm that the development of students' skills in working with mathematical texts requires specially organized learning activities aimed at developing various aspects of understanding mathematical information.

The developed model may serve as a theoretical basis for designing a methodology for teaching mathematical analysis aimed at forming the following skills in students:

- analyzing the structure of mathematical text;
- identifying semantic relationships between mathematical concepts;
- interpreting various forms of representation of mathematical information;
- constructing conceptual schemes of mathematical knowledge.

The development of these skills contributes to students' ability to work independently with mathematical texts and to acquire mathematical knowledge more effectively.

6. Conclusion

This article presents a three-layer model for understanding mathematical text based on the integration of thesaurus, semantic, and semiotic approaches.

The proposed model shows that understanding mathematical text is formed through:

- interpretation of the forms of representation of mathematical information;
- identification of logical relationships between concepts;
- formation of a system of mathematical knowledge.

The scientific novelty of the study is as follows.

An integrative model for understanding mathematical text was developed based on the combination of three cognitive components involved in understanding mathematical information: semiotic, semantic, and thesaurus-based.

It is shown that understanding mathematical text is formed through the interaction of three cognitive processes:

- interpretation of symbolic forms of representation of mathematical information;
- identification of meaningful relationships between mathematical concepts;

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- formation of a system of mathematical knowledge.

A structure for analyzing mathematical text is proposed that makes it possible to identify semantic relations between mathematical concepts and use them in the process of teaching mathematical analysis.

It is substantiated that the proposed model can be used as a methodological basis for developing students' skills in working with mathematical information.

The results of the study can be used in developing methodologies for teaching mathematical analysis, creating learning tasks aimed at developing skills in working with mathematical texts, and designing diagnostic materials.

References

1. Arnon, I., Cottrill, J., Dubinsky, E., Oktaç, A., Roa Fuentes, S., Trigueros, M., & Weller, K. (2014). *APOS theory: A framework for research and curriculum development in mathematics education*. Springer.

2. Borasi, R., & Siegel, M. (2020). Reading mathematics texts. *Mathematics Education Research Journal*.

3. Duval, R. (2006). A cognitive analysis of problems of comprehension in learning mathematics. *Educational Studies in Mathematics, 61*(1–2), 103–131.

4. Harel, G., & Koichu, B. (2021). *Learning and teaching mathematical reasoning*. Cambridge University Press.

5. Inglis, M., & Alcock, L. (2018). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education*.

6. Lithner, J. (2017). Mathematical reasoning in task solving. *Educational Studies in Mathematics*.

7. Mesa, V., & Griffiths, B. (2020). Reading mathematics in undergraduate studies.

8. Selden, A., & Selden, J. (2017). Teaching proving. *International Journal of Research in Undergraduate Mathematics Education*.

9. Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

10. Sowa, J. F. (2000). *Knowledge representation: Logical, philosophical, and computational foundations*. Brooks/Cole.

11. Tall, D. (2013). *How humans learn to think mathematically*. Cambridge University Press.

12. Turgunbayev, R. M. (2022). Thesaurus approach in the methodology of teaching mathematics. *Physics, Mathematics, Informatics*, (3), 90–97.

13. Turgunbayev, R. M., & Umaraliyeva, D. U. (2023). Method of questions in teaching students to work with mathematical text. *Physics, Mathematics, Informatics*, (6), 30–37.

14. Weber, K., & Mejía-Ramos, J. (2019). The development of mathematical reasoning. *Educational Studies in Mathematics*.

15. Naykhanova, L. V. (2008). Main types of semantic relations between terms of a subject domain. (1), 62–71.

16. Turgunbayev, R. M. (2021). Formation of the educational thesaurus of mathematical analysis and its significance. *Teacher and Continuous Education*, (1), 127–132.

 ISSN: 2786-9458